Long-Boom Yagi Rules of Thumb:
A Comparison with Modeling Data

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Anyone interested in long-boom Yagis can hardly escape encountering any number of small equations for calculating some Yagi parameters. I call them "rules of thumb" or approximations, but many individuals have, from time to time, treated them as being precise methods of determining the gain or beamwidth of a Yagi. Because these equations have acquired a life of their own over time, I became curious about their adequacy as a guide to Yagi performance aspects.

The following notes will review a few of the rules of thumb by comparing the calculations with the results of NEC-4 modeling. Over time, I have collected many dozens of Yagi models, but three sets have special relevance to this inquiry. All three Yagis sets use 432 MHz as the design frequency, all 4 uses 4-mm diameter elements, and all three form sequences. The first set includes DL6WU Yagis from 10 through 40 elements on booms from 2 λ to 14 λ long. The second group consists of Yagis optimized by Dean Straw, N6BV. The boomlengths are virtually the same as those in the DL6WU series for each element count. However, the N6BV set consists of individually optimized arrays. The final set of test Yagis uses closer spacing and different algorithms than the DL6WU sequence, resulting in a series from 10 to 50 elements on booms from 1.5λ to 14.5 λ. **Fig. 1** compares the outlines of the longest members of the 3 sets of Yagis.

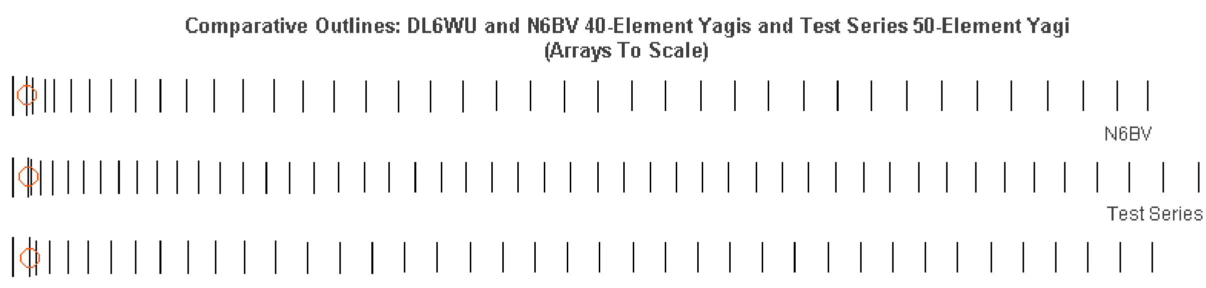


Fig. 1

This initial note will examine several rules of thumb for gain. One collection is based on the horizontal and vertical beamwidth of the Yagi, while another technique bases the gain calculation on boomlength. I shall also look at some calculations of horizontal beamwidth based on gain and a calculation of vertical beamwidth from the horizontal beamwidth. The collection is far from exhaustive of the available rules of thumb, but it may be enough for a start.

Basic Yagi Performance

In the course of a study on the stacking behavior of long-boom Yagis, I gathered a considerable mass of data on trimming Yagi series, most notably, the classic DL6WU design. Using a set of algorithms, the series produces designs that differ in element length and spacing only as we add new directors. Otherwise put, to produce a shorter Yagi, we simply trim away the most forward directors down to the length that we want. The program DL6WU-GG.EXE includes the algorithms for designing almost any length Yagi from 10 elements onward. I chose to stop at 40 elements. The program is available from the web site maintained by Ian White, G3SEK. See http://www.ifwtech.co.uk/g3sek. The program actually comes in two parts zipped together. Accompanying the DOS program is a text containing the original Basic code, as revised through 2003. The dimensions for the set of Yagis used here appear in my notes on trimming Yagis at my site: http://www.cebik.com/yagitrim.pdf (with the frequency sweep graphs in a separate document, http://www.cebik.com/yagitrim2.pdf).

Table 1 provides a summary of the modeled free-space performance of the Yagis in the DL6WU series. Most of the data not relevant to these notes has been omitted.

DL6WU 40-Element Series							Table 1
Modeled Single-Unit Free-Space Performance at 432 MHz							
Elements	Bm-Ln	Gain	180 F-B	H BW	H F/SL	V BW	V F/SL
10	2.145068	13.88	31.92	37.6	19.08	41.6	13.4
11	2.490042	14.26	16.96	35.8	18.19	39	13.32
12	2.850003	14.76	15.49	34	17.81	36.6	13.63
13	3.224951	15.28	19.31	32.4	17.82	34.8	14.14
14	3.615028	15.69	28.63	31.2	17.78	33.2	14.5
15	4.015049	15.99	21.14	30	17.58	31.8	14.62
16	4.41507	16.27	17.54	29	17.4	30.6	14.69
17	4.814946	16.59	17.68	28	17.32	29.5	14.88
18	5.214967	16.92	21.43	27.2	17.37	28.6	15.08
19	5.614988	17.21	33.2	26.6	17.33	27.6	15.19
20	6.015008	17.43	24.37	25.8	17.16	26.8	15.17
21	6.415029	17.61	19.8	25.2	16.96	26.2	15.11
22	6.81505	17.82	19.51	24.6	16.85	25.6	15.11
23	7.215071	18.06	22.92	24	16.77	24.8	15.15
24	7.614947	18.28	35.99	23.6	16.68	24.4	15.17
25	8.014968	18.45	26.69	23	16.57	23.8	15.08
26	8.414989	18.59	21.48	22.6	16.38	23.4	14.99
27	8.815009	18.74	20.94	22.2	16.32	22.8	15.02
28	9.21503	18.92	24.14	21.8	16.24	22.4	14.96
29	9.61505	19.09	37.43	21.4	16.16	22	14.96
30	10.01507	19.23	28.42	21.2	16.04	21.6	14.86
31	10.41495	19.35	22.83	20.8	15.98	21.2	14.88
32	10.81497	19.47	22.14	20.4	15.88	21	14.79
33	11.21499	19.61	25.2	20.2	15.91	20.6	14.84
34	11.61501	19.75	38.02	20	15.76	20.4	14.75
35	12.01503	19.87	29.7	19.6	15.72	20	14.72
36	12.41505	19.97	23.93	19.4	15.67	19.8	14.75
37	12.81507	20.07	23.15	19	15.58	19.6	14.66
38	13.21495	20.18	26.12	18.8	15.57	19.2	14.66
39	13.61497	20.3	38.19	18.6	15.55	19	14.71
40	14.01499	20.4	30.61	18.4	15.43	18.8	14.6

The N6BV series uses the element count and the boomlength (within about 0.1 λ) of the DL6WU wide-band Yagis. By reducing the operating bandwidth and subjecting each Yagi to computer optimization, N6BV created a series of individual Yagis. The optimizing process improves both gain and front-to-back ratio, while preserving a low 50-Ω SWR, at least within the more restricted passband. However, each Yagi in the sequence now has a unique set of values for element length and spacing. **Table 2** summarizes the performance of the Yagis in the N6BV series.

N6BV 40-Element Series			BV1070-1 through BV4070-1 Series				Table 2
Modeled Single-Unit Free-Space Performance at 432 MHz							
Elements	Bm-Ln	Gain	180 F-B	H BW	H F/SL	V BW	V F/SL
10	2.00048	13.64	31.7	38	18.54	42	12.57
11	2.3755	14.27	20.9	36.4	19.99	40	14.76
12	2.77702	14.8	24.64	34	16.89	36.7	12.57
13	3.1625	15.28	22.22	32.8	18.11	35.2	14.31
14	3.57397	15.76	26.96	31	16.87	33	13.57
15	3.89277	16.08	28.54	29	14.88	30.6	11.99
16	4.29392	16.42	29.32	28.4	16.09	30	13.49
17	4.74347	16.48	22.49	28.8	17.65	30.4	15.09
18	5.10764	16.85	24.59	27.8	16.92	29.2	14.54
19	5.50513	17.21	29.13	26.4	15.67	27.6	13.55
20	5.90818	17.53	34.42	25.2	15.1	26.2	13.24
21	6.31636	17.82	36.09	24.2	14.88	25.2	13.14
22	6.72615	18.11	33.78	23.4	15.25	24.2	13.64
23	7.1142	18.26	27.13	23.2	15.97	24	14.46
24	7.51183	18.38	29.19	23.2	15.36	24	13.87
25	7.91006	18.63	37.24	22.2	14.61	22.8	13.21
26	8.33976	18.89	39.3	21.4	14.44	22	13.15
27	8.74875	19.09	35.51	21	14.45	21.6	13.18
28	9.10223	19.28	31.69	20.5	14.96	21	13.79
29	9.47776	19.39	34.7	20.6	14.91	21	13.74
30	9.90051	19.65	34.18	19.6	14.96	20.2	13.89
31	10.3143	19.67	38.8	19.8	14.84	20.2	13.76
32	10.6869	19.72	35.64	19.2	14.55	19.8	13.56
33	11.0886	20.12	32.94	18.8	16.11	19.2	15.16
34	11.4812	20.17	35.34	18.6	15.12	19	14.22
35	11.9248	20.4	31.77	17.8	14.65	18.2	13.83
36	12.2971	20.4	36.4	18.2	14.76	18.4	13.87
37	12.7639	20.61	41.55	17.4	14.66	17.8	13.85
38	13.1037	20.69	33.58	17.4	14.81	17.8	14
39	13.4997	20.82	36.33	17.2	15.13	17.4	14.39
40	13.9457	21.06	34.97	16.8	15.32	17.2	14.58

The final set of Yagis derives ultimately from a design by David Tanner, VK3AUU. However, I revised the element diameter and the element length algorithms to bring the frequency of maximum gain closer to the design frequency, yielding higher gain, but somewhat lesser front-to-sidelobe performance. The design uses a variable spacing algorithm that grows larger with a longer boomlength (in contrast to the constant spacing used by DL6WU Yagis from 12 elements onward). Hence, the series runs from 10 to 50 elements. The final algorithm for element length does not yield a linear outline to the elements of the longest version. Despite the more complex curvature of the element lengths, the result is a second trimming Yagi series. To create a shorter Yagi, simply remove the most forward directors from the longest version until you reach the desired boomlength. The performance of these Yagis is also part of the data in the documents previously noted for the DL6WU sequence.

Table 3 summarizes the relevant modeled free-space performance of the series. The designation "LB" abbreviates "long boom" and is not my initials.

LB10-LB50 Yagi Series							Table 3
Modeled Single-Unit Free-Space Performance at 432 MHz							
Elements	Bm-Ln	Gain	180 F-B	H BW	H F/SL	V BW	V F/SL
10	1.52962	12.7	17.6	41	19.65	46.4	12.41
11	1.77685	13.28	16.96	38.6	18.23	43	12.43
12	2.02258	13.64	19.6	37.6	18	41	12.73
13	2.28	14.08	17.04	35.4	17.21	38.8	12.71
14	2.54783	14.46	25.31	34.4	17.31	37.2	13.22
15	2.82488	14.79	16.19	33	16.9	35.6	13.27
16	3.11004	15.18	34.39	32	17.15	34.4	13.84
17	3.40226	15.45	18.46	31	17.17	33.2	14.12
18	3.70057	15.78	21.34	30.2	17.3	32	14.53
19	4.00408	16.08	25.33	29.4	17.66	31.2	15.06
20	4.31197	16.32	18.76	28.6	17.73	30.2	15.31
21	4.6235	16.61	28.14	28	18.07	29.4	15.8
22	4.93798	16.85	22.1	27.2	18.29	28.6	16.16
23	5.25483	17.07	20.12	26.6	18.37	27.8	16.39
24	5.57352	17.32	36.5	26	18.64	27.2	16.78
25	5.89361	17.52	21.34	25.4	18.68	26.6	16.88
26	6.21472	17.72	21.56	24.8	18.67	25.8	16.98
27	6.53657	17.94	37.54	24.4	18.73	25.2	17.15
28	6.85891	18.11	21.18	23.8	18.54	24.8	17.01
29	7.1816	18.29	22.97	23.4	18.42	24.2	16.98
30	7.50457	18.49	33.15	23	18.3	23.6	16.88
31	7.82782	18.64	21.23	22.4	17.98	23.2	16.65
32	8.15142	18.81	24.43	22	17.83	22.6	16.51
33	8.47553	18.98	30.48	21.6	17.53	22.2	16.29
34	8.80036	19.11	21.4	21.2	17.22	21.8	16
35	9.12621	19.26	25.69	20.8	16.98	21.4	15.84
36	9.45346	19.42	29.4	20.5	16.7	21	15.56
37	9.78155	19.54	21.68	20.2	16.43	20.6	15.37
38	10.114	19.68	26.02	19.8	16.17	20.2	15.12
39	10.4484	19.83	30.71	19.4	15.97	20	14.92
40	10.7865	19.94	22.09	19.2	15.67	19.6	14.7
41	11.1289	20.06	24.66	18.8	15.48	19.2	14.51
42	11.4766	20.2	40.83	18.6	15.35	19	14.39
43	11.8303	20.32	23.57	18.4	15.08	18.6	14.16
44	12.1911	20.42	22.45	18	14.92	18.4	14.03
45	12.56	20.54	28.54	17.8	14.86	18.2	13.97
46	12.9381	20.67	34.4	17.6	14.66	18	13.84
47	13.3266	20.78	23.92	17.4	14.53	17.6	13.71
48	13.7268	20.88	21.71	17.2	14.5	17.4	13.68
49	14.1401	20.99	22.34	17	14.43	17.2	13.68
50	14.5677	21.1	24.84	16.8	14.37	17	13.62

These three tables comprise essential data for comparison with the results of the rule-of-thumb equations. I shall present several other tables. Each is a continuation to the right of the first three tables. The nature of presentation space makes it impossible to place all the tables together side-by-side. More significantly, the tables contain data on enough Yagis (102) to form a quite reasonable test of the rule-of-thumb equations.

Gain from Beamwidth

Of all of the rules of thumb, one that calculates Yagi gain from the horizontal and vertical (or, more properly, the E-plane and H-plane) beamwidths is perhaps the most venerable. Deriving from Kraus, the equation has many forms. For example, one version appears in the *Beam Antenna Handbook* by Orr and Cowan, page 47.

$$G_{dBi} = 10\log_{10}\left(\frac{52525}{HBW \cdot VBW}\right) \quad \text{Orr/Cowan}$$

A different version, directly attributed to Kraus, appears in the RSGB volume, *The VHF/UHF DX Book*, page 7-5 (in the 2nd edition).

$$G_{dBi} = 10\log_{10}\left(\frac{42000}{HBW \cdot VBW}\right) \quad \text{RSGB}$$

The version that appears in *Antennas* by Krause (page 99 in the 2nd edition) differs from either of the first two versions.

$$G_{dBi} = 10\log_{10}\left(\frac{41000}{HBW \cdot VBW}\right) \quad \text{Kraus}$$

All three versions use the necessary adjustments to result in gain values in dBi, facilitating comparison with the modeled data. In fact, the only difference among the 3 versions is the value of the numerator. Kraus himself notes that between the first and second editions of *Antennas*, a number of individuals suggested additional (and smaller) values for the numerator, which represents the number of square degrees such that a beam with a 1° pencil beam would have a gain of about 46 dBi. (Note: $360^{\circ 2}/\pi$ = 41,253 square degrees.)

The question at hand is how well these equations approximate the results of NEC-4 modeling. All of the models in the previous tables use a free-space environment with aluminum elements. Hence, all of the models provide data for the horizontal and vertical beamwidths. They also show a degree of loss based on the material conductivity of the elements within a parasitic element design.

We may look at the data in two ways. Graphically, we can crate data curves to show the general trends, with each curve extending from the shortest to the longest Yagi in a series. These results are useful for seeing the general properties of the information. We may also look at data tables to extract other information, such as the percentage of error between a calculated and a modeled result. Such data does not graph as well, but table scanning produces trends of its own. In all cases, the use of these equations rests on the initially modeled horizontal and vertical beamwidths in the initial tables.

Fig. 2 and **Table 4** provide data for the DL6WU series. **Fig. 3** and **Table 5** show comparable data for the N6BV Yagi sequence. The test or "LB" series data appears in **Fig. 4** and **Table 6**.

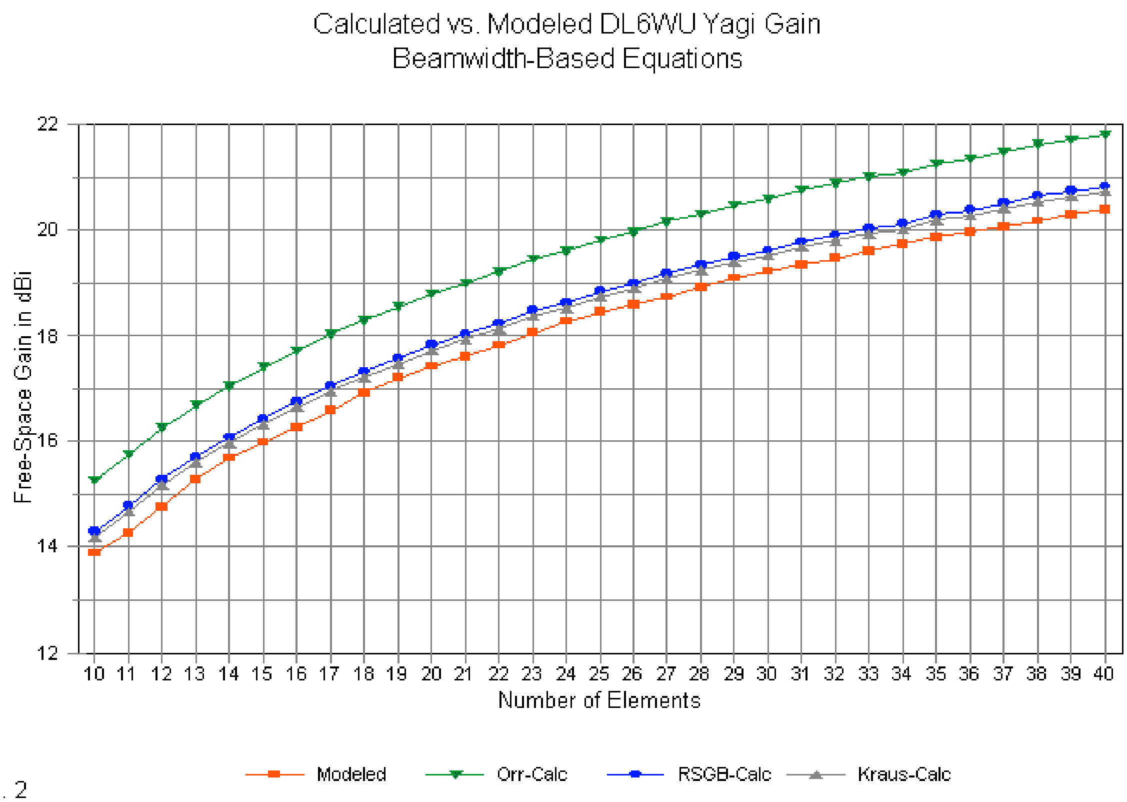
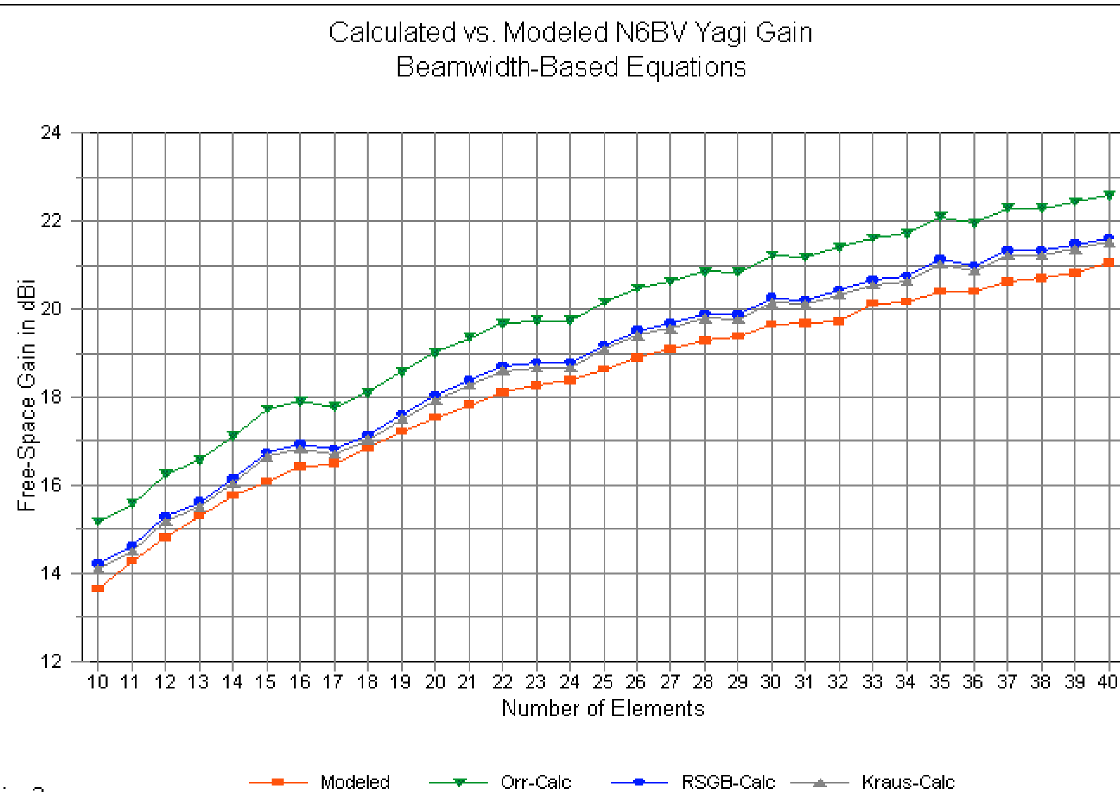

Calculated vs. Modeled DL6WU Yagi Gain
Beamwidth-Based Equations
Free-Space Gain in dBi
Number of Elements
Modeled
Orr-Calc
RSGB-Calc
Kraus-Calc

Fig. 2


Calculated vs. Modeled N6BV Yagi Gain
Beamwidth-Based Equations
Free-Space Gain in dBi
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Kraus-Calc

Fig. 3

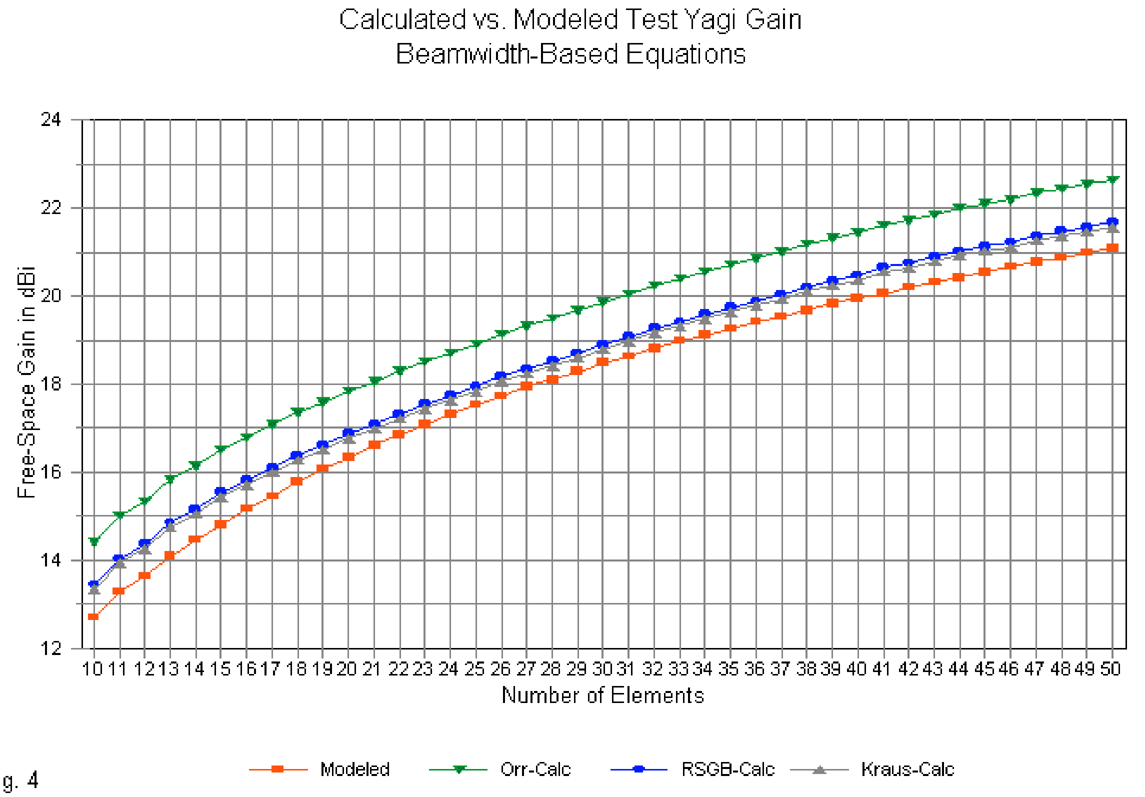


Fig. 4

The DL6WU and test ("LB") Yagi series produce smooth curves throughout. The N6BV series yields somewhat more erratic curves as a function of the individualized optimization of each Yagi in that sequence.

More significantly, the modeled performance of the Yagis in all three sets shows a lower gain than predicted by simple application of the basic equation in any of its forms. The RSGB and Kraus versions differ from each other only by a small amount, but both estimate high. Kraus was aware of this, since the two correctives that he introduced (beam efficiency and pattern factor) have the effect of reducing the numerator. Indeed, he appeared to be nonplused by the fact that numerous readers took his original equation as yielding more accurate results than what he terms "ballpark" values (page 100). He notes specifically that "(1) the effect of the minor lobes is neglected, (2) the angle product may not be rigorously related to the true solid angle of the main beam and (3) the angle product relation to the true solid angle varies according to the type of antenna pattern involved." The arrays under scrutiny do have significant minor lobes, especially the first forward sidelobes in the E-plane and many of the sidelobes in the H-plane.

The question that remains is how big a ballpark we have. **Table 4**, **5**, and **6**, one for each Yagi series, list the calculated gain for the array according to the 3 variations in the equation. Accompanying these data columns is a calculation of the percentage of error in the calculation, using the modeled gain value as the baseline. The modeled gain appears in the earlier tables.

DL6WU 40-Element Series			Gain Calculations					Table 4
				Percentage Error				
Elements	Orr	RSGB	Kraus	Orr	RSGB	Kraus	BL-Eq	% Error
10	15.26085	14.28968	14.18503	9.948479	2.951593	2.1976	13.73524	1.053929
11	15.75418	14.78302	14.67836	10.47815	3.667718	2.933816	14.24041	0.137552
12	16.25406	15.28289	15.17824	10.12236	3.542635	2.833594	14.69779	0.423237
13	16.68242	15.71125	15.6066	9.178129	2.822319	2.137409	15.11648	1.081731
14	17.05073	16.07957	15.97491	8.672618	2.482894	1.815881	15.50327	1.204449
15	17.40818	16.43701	16.33235	8.869149	2.795554	2.141056	15.85879	0.827373
16	17.72247	16.7513	16.64664	8.927267	2.958197	2.314962	16.18051	0.553053
17	18.03386	17.06269	16.95804	8.703196	2.849261	2.218433	16.47421	0.702837
18	18.29431	17.32314	17.21849	8.122407	2.382645	1.764121	16.74456	1.04773
19	18.54575	17.57459	17.46993	7.761496	2.118453	1.510351	16.99492	1.265549
20	18.80612	17.83495	17.73029	7.895098	2.323281	1.722855	17.22804	1.172261
21	19.00664	18.03547	17.93082	7.930961	2.416096	1.821807	17.44615	0.93918
22	19.21191	18.24074	18.13609	7.810942	2.361067	1.773781	17.65106	0.957118
23	19.45703	18.48586	18.38121	7.7355	2.358049	1.778568	17.84428	1.208921
24	19.60064	18.62947	18.52482	7.22452	1.911787	1.339279	18.027	1.403441
25	19.82061	18.84944	18.74479	7.428795	2.165013	1.597781	18.20043	1.371209
26	19.97042	18.99925	18.8946	7.425592	2.201452	1.638492	18.36542	1.222854
27	20.16078	19.18961	19.08496	7.58155	2.399225	1.84077	18.52274	1.17295
28	20.31662	19.34545	19.24079	7.381689	2.248667	1.695526	18.67307	1.32236
29	20.4753	19.50413	19.39947	7.256658	2.169347	1.621132	18.81702	1.450697
30	20.59576	19.6246	19.51994	7.102259	2.051985	1.507761	18.9551	1.450263
31	20.75967	19.7885	19.68385	7.285109	2.266155	1.725305	19.08773	1.374045
32	20.88517	19.914	19.80934	7.268444	2.280423	1.742907	19.2154	1.324992
33	21.01147	20.04031	19.93565	7.146735	2.194324	1.660646	19.33843	1.404295
34	21.09706	20.12589	20.02124	6.820552	1.903247	1.373352	19.45715	1.505088
35	21.2708	20.29963	20.19498	7.049824	2.162216	1.63552	19.57185	1.523341
36	21.35899	20.38782	20.28317	6.95539	2.092257	1.568199	19.6828	1.459152
37	21.49356	20.5224	20.41774	7.092994	2.254092	1.732645	19.79022	1.41371
38	21.62907	20.6579	20.55325	7.180723	2.368197	1.849593	19.89431	1.436033
39	21.721	20.74983	20.64517	6.999976	2.215899	1.70036	19.99533	1.523705
40	21.8139	20.84274	20.73808	6.930901	2.170275	1.657264	20.09342	1.525754

Note that the Kraus ballpark value is never more than 3% high for the DL6WU Yagi series in **Table 4**. In fact, it reaches its closest approach to the modeled values at 24 elements, which corresponds to a 7.6 λ boom. At this length, even using lossless elements would not have raised the gain value more than 20% of the way to the Kraus equation value. However, accounting for the strong minor lobes of the array might well have done the job. Although the horizontal sidelobe rejection is not maximum at this boomlength, the vertical sidelobe rejection (or ratio to the main lobe) is close to its peak value.

Table 5 for the N6BV series of Yagis shows a similar phenomenon, despite the more erratic set of values resulting from individual optimizing. The closest approach of the Kraus calculation to the modeled results occurs at 18 elements or a boom that is 5.1 λ long. The front-to-sidelobe ratios for both the horizontal and vertical patterns are one step removed from their peak values. Nevertheless, all of the equation-based results produce differentials from the modeled results ranging from 1 to 3 percent, and all are high. The Orr and RSGB equations yield results proportionately higher.

N6BV 40-Element Series			Gain Calculations					Table 5
				Percentage Error				
Elements	Orr	RSGB	Kraus	Orr	RSGB	Kraus	BL-Eq	% Error
10	15.17333	14.20216	14.09751	11.24144	4.121437	3.354177	13.49885	1.045668
11	15.57205	14.60088	14.49622	9.124365	2.318705	1.585318	14.08089	1.343031
12	16.24221	15.27104	15.16639	9.744668	3.182724	2.4756	14.60992	1.30106
13	16.5795	15.60833	15.50367	8.504552	2.148742	1.463832	15.05024	1.526633
14	17.1049	16.13374	16.02908	8.533657	2.371425	1.707375	15.46458	1.91032
15	17.72247	16.7513	16.64664	10.21434	4.174743	3.523907	15.75402	2.069198
16	17.89926	16.9281	16.82344	9.00892	3.094379	2.45702	16.08626	2.074685
17	17.781	16.80983	16.70518	7.894417	2.001409	1.36637	16.42355	0.343714
18	18.10938	17.13822	17.03356	7.47409	1.710483	1.089389	16.67412	1.054819
19	18.57853	17.60736	17.50271	7.95195	2.308907	1.700805	16.92799	1.665956
20	19.00664	18.03547	17.93082	8.423516	2.883483	2.286482	17.16734	2.112506
21	19.3515	18.38033	18.27568	8.594285	3.14441	2.557124	17.39364	2.451231
22	19.67335	18.70218	18.59753	8.632515	3.26991	2.692028	17.60658	2.859277
23	19.74667	18.7755	18.67085	8.141667	2.823114	2.24998	17.79658	2.603964
24	19.74667	18.7755	18.67085	7.435628	2.151799	1.582406	17.98082	2.22005
25	20.16078	19.18961	19.08496	8.21676	3.003836	2.442085	18.1558	2.611825
26	20.4753	19.50413	19.39947	8.39225	3.251077	2.697057	18.335	3.02701
27	20.63693	19.66576	19.56111	8.103353	3.016042	2.467827	18.49718	3.204927
28	20.86393	19.89276	19.78811	8.2154	3.178223	2.63541	18.63135	3.481482
29	20.8428	19.87163	19.76697	7.492499	2.483898	1.944164	18.7683	3.312475
30	21.22759	20.25642	20.15176	8.028429	3.086099	2.553507	18.91613	3.879604
31	21.1835	20.21233	20.10767	7.694433	2.757129	2.225079	19.05483	3.22842
32	21.404	20.43283	20.32817	8.539536	3.61475	3.084049	19.17504	2.842009
33	21.62907	20.6579	20.55325	7.500347	2.67347	2.153319	19.30004	4.248497
34	21.721	20.74983	20.64517	7.689614	2.874702	2.355841	19.4179	3.873226
35	22.09875	21.12758	21.02292	8.32719	3.566564	3.053552	19.54632	4.367479
36	21.95477	20.9836	20.87895	7.621414	2.860788	2.347777	19.65046	3.814358
37	22.29397	21.3228	21.21815	8.170636	3.458517	2.950733	19.77667	4.2137
38	22.29397	21.3228	21.21815	7.752383	3.058484	2.552663	19.86567	4.149506
39	22.44288	21.47172	21.36706	7.79483	3.13024	2.627577	19.96653	4.274513
40	22.59528	21.62412	21.51946	7.290044	2.678612	2.181677	20.07663	4.89806

The test Yagi series, in **Table 6**, is similar to the DL6WU series in producing the closest approach of the Kraus-calculated gain to the modeled gain with a boomlength of 7.5 λ. This length corresponds to 30 elements in the series. Unfortunately for prospective trends based on front-to-sidelobe ratio, this element count does not produce the best values in that performance category.

Overall, the error range for the Kraus equation relative to the modeled gain values runs from 1.5 to 5 percent, with the other equations yielding more distant results. The DL6WU series error values above the boomlength of closest approach tend to fluctuate in a very small way at a value about 0.5 percent above the best value. However, the test series shows as much as a full percent higher error for longer booms. Part of the variation may lie in the more complex algorithms used for both element spacing and length.

Perhaps the variations in the three test-series results lend some credence to Kraus' own note that " the angle product may not be rigorously related to the true solid angle of the main beam." Still, the Kraus version of the equation does provide a reasonable ballpark value for array gain. However, if the calculated value rests on the modeled E-plane and H-plane beamwidths, then the equation becomes unnecessary, since the same NEC calculations yield the array gain as well.

LB10-LB50 Yagi Series			Gain Calculations					Table 6
	52525	42000	41000	Percentage Error				
Elements	Orr	RSGB	Kraus	Orr	RSGB	Kraus	BL-Eq	% Error
10	14.41064	13.43947	13.33482	13.46962	5.822634	4.998584	12.58975	0.875699
11	15.0031	14.03194	13.92728	12.97517	5.662163	4.874104	13.09728	1.39513
12	15.32394	14.35278	14.24812	12.34563	5.22563	4.458369	13.53606	0.767841
13	15.82531	14.85414	14.74949	12.39567	5.498175	4.754891	13.94189	0.990599
14	16.13265	15.16148	15.05682	11.56741	4.851169	4.127419	14.31813	0.990845
15	16.50402	15.53285	15.4282	11.58905	5.022674	4.315072	14.6678	0.833117
16	16.78658	15.81541	15.71075	10.58351	4.185828	3.496406	14.99357	1.243369
17	17.07866	16.1075	16.00284	10.54151	4.255632	3.578258	15.29779	0.995003
18	17.35209	16.38092	16.27627	9.962556	3.808135	3.144926	15.5825	1.267478
19	17.57864	16.60747	16.50282	9.319909	3.280309	2.629473	15.84952	1.454167
20	17.83993	16.86876	16.76411	9.313302	3.362519	2.721255	16.10047	1.363496
21	18.04861	17.07744	16.97278	8.66109	2.814204	2.184136	16.33677	1.672468
22	18.29431	17.32314	17.21849	8.57158	2.807973	2.186879	16.55968	1.753145
23	18.5144	17.54323	17.43857	8.461607	2.772282	2.159193	16.77036	1.786738
24	18.70824	17.73707	17.63242	8.015231	2.408028	1.803788	16.96981	2.063602
25	18.90651	17.93534	17.83069	7.913853	2.370659	1.773316	17.15897	2.104002
26	19.14295	18.17178	18.06712	8.030173	2.549543	1.958943	17.33869	2.199196
27	19.31576	18.34459	18.23993	7.668656	2.255235	1.671878	17.50973	2.457323
28	19.49337	18.52221	18.41755	7.638731	2.276127	1.698245	17.67279	2.473917
29	19.67335	18.70218	18.59753	7.563414	2.253585	1.681391	17.82853	2.588406
30	19.85726	18.88609	18.78144	7.394604	2.142209	1.576204	17.97754	2.850549
31	20.0463	19.07513	18.97048	7.544531	2.334404	1.772954	18.1204	2.867495
32	20.23835	19.26718	19.16253	7.593564	2.430525	1.874149	18.25762	3.025479
33	20.39559	19.42443	19.31977	7.458342	2.341547	1.790154	18.3897	3.20994
34	20.55574	19.58457	19.47992	7.565343	2.483356	1.935714	18.5171	3.201886
35	20.71889	19.74772	19.64307	7.574712	2.532304	1.988928	18.64027	3.324708
36	20.86393	19.89276	19.78811	7.435268	2.434404	1.895505	18.75961	3.520286
37	21.01147	20.04031	19.93565	7.530577	2.560425	2.024835	18.87518	3.522192
38	21.1835	20.21233	20.10767	7.63971	2.704915	2.173135	18.9884	3.642229
39	21.31534	20.34418	20.23952	7.490385	2.592918	2.06516	19.09859	3.829663
40	21.44809	20.47692	20.37227	7.563127	2.692678	2.167831	19.20647	3.819191
41	21.62907	20.6579	20.55325	7.821884	2.980569	2.458862	19.31233	3.871478
42	21.721	20.74983	20.64517	7.529679	2.721918	2.203827	19.41654	4.034996
43	21.86035	20.88919	20.78453	7.580477	2.801108	2.286077	19.51937	4.101737
44	22.00276	21.03159	20.92694	7.751015	2.995052	2.482543	19.62113	4.071454
45	22.09875	21.12758	21.02292	7.588835	2.860657	2.351142	19.72212	4.147023
46	22.19581	21.22464	21.11999	7.381756	2.683315	2.177005	19.82259	4.274972
47	22.34304	21.37187	21.26722	7.521855	2.848286	2.344655	19.92281	4.30255
48	22.44288	21.47172	21.36706	7.485075	2.833889	2.332671	20.02304	4.279867
49	22.54389	21.57272	21.46806	7.402987	2.776175	2.277584	20.12353	4.305759
50	22.64608	21.67491	21.57026	7.327387	2.724696	2.228704	20.22445	4.329167

Gain from Boomlength

A second method of calculating gain derives from the array boomlength and appears in the final two columns of **Table 4**, **5**, and **6** as "BL-Eq" and "% Error." The RSGB handbook also presents this equation on page 7-16. Initially specified in terms of a dBd gain value, I have adjusted it for gain values in dBi.

$$G_{dBi} = 7.8 \log_{10}\left(\frac{L}{\lambda}\right) + 11.15$$

Interestingly, the equation appears to come closest to modeled gain values for the shortest boomlengths in each sequence.

The new equation appears to work best for the DL6WU series of Yagis. This result is not surprising, since Gunter Hoch is the listed author of the chapter in the RSGB handbook. However, for the N6BV and the test Yagi series, the departure from modeled results grows to between 4 and 5 percent for the longest booms (or highest element counts). The following graphs, beginning with the DL6WU Yagis in **Fig. 5**, clearly show the trends. The N6BV series in **Fig. 6** is interesting because a boomlength-based equation produces a smooth gain curve, in contrast to the more erratic curve yielded by the individually optimized beams when modeled. The test-series data, graphed in **Fig. 7**, return the lines to a pair of smooth curves. In all cases, the modeled gain is higher than the calculated gain. Unfortunately, there appears to be no single adjustment to the equation that will satisfy all three series of modeled results. Like the Kraus equation, the results are ballpark values that may be most useful for shorter boomlengths in the test sequences of Yagis. Because the equation requires only a boomlength--either actual or projected--as its basis and because it calculates gain on the conservative side, it may be most useful as a preliminary planning tool. If you need a certain gain to effect a certain communications circuit, then using the equation will generally guarantee that a reasonable Yagi design for the boomlength will achieve the necessary gain, with just a bit to spare

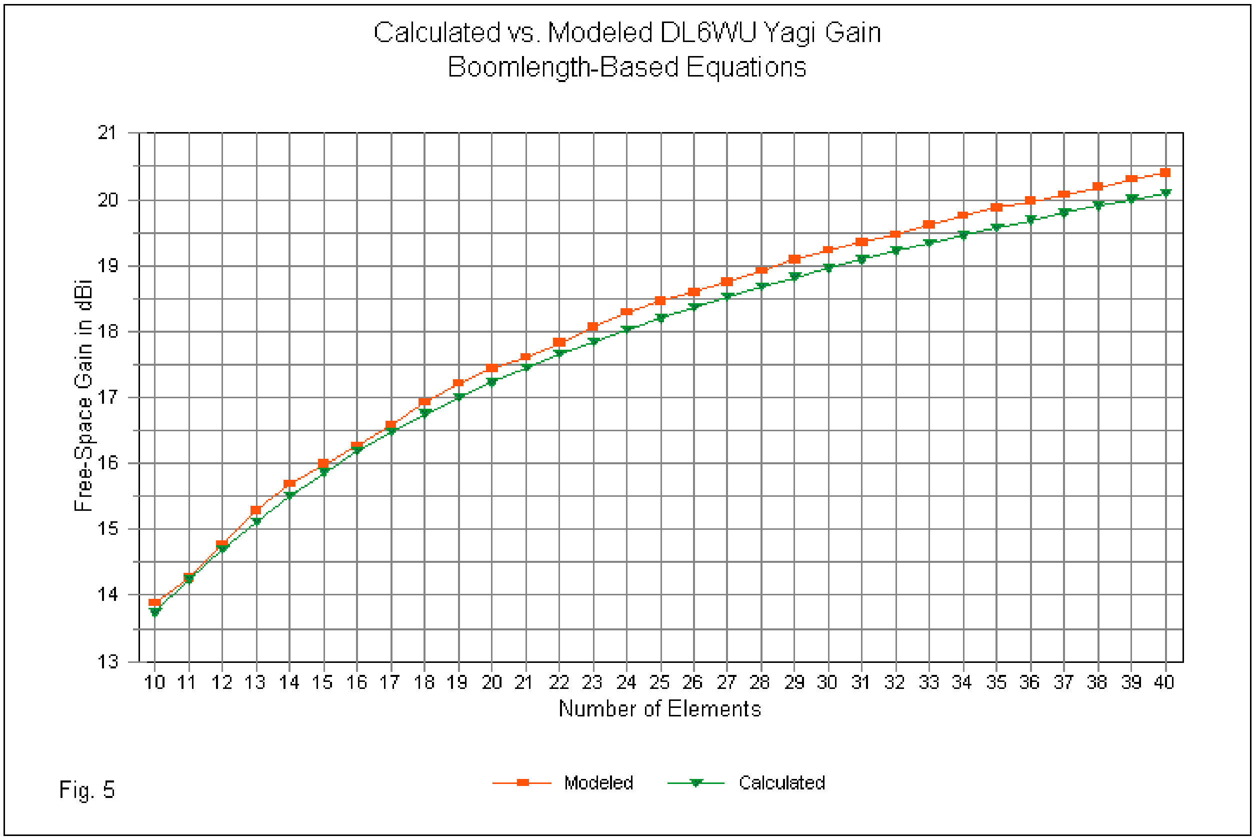


Fig. 5

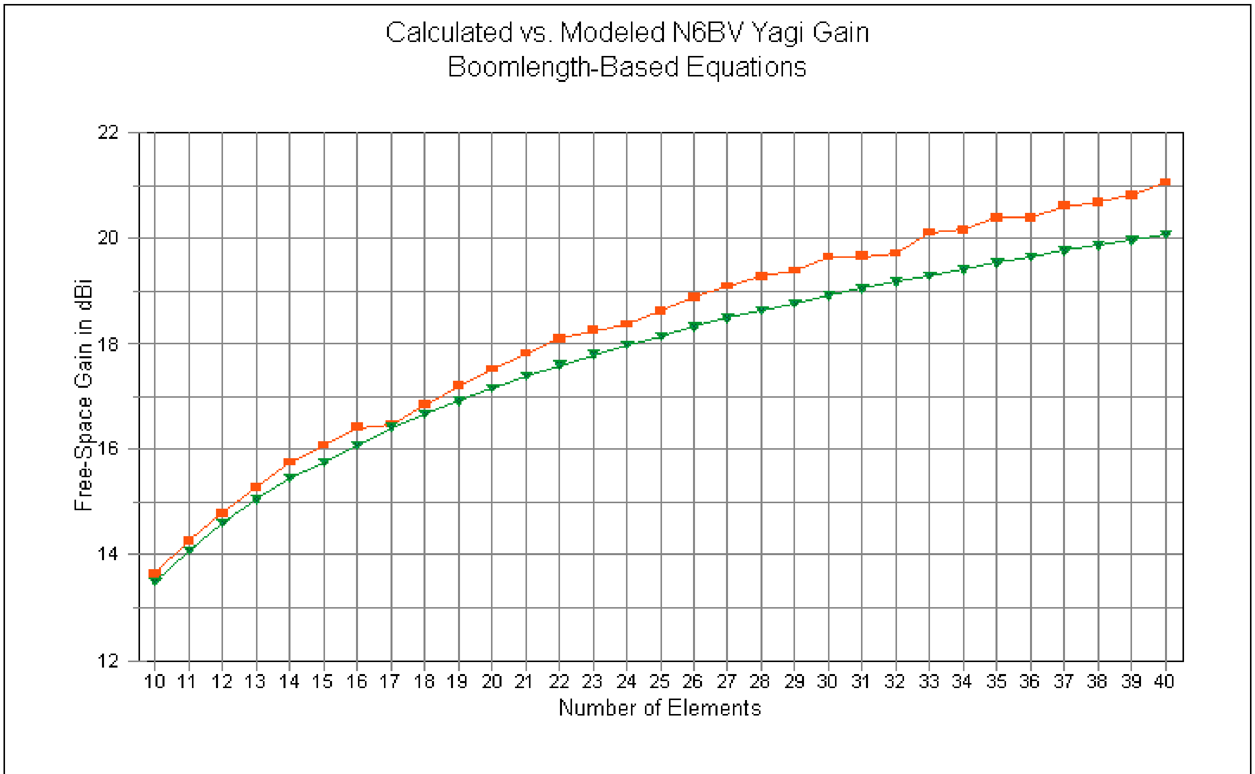

Calculated vs. Modeled N6BV Yagi Gain
Boomlength-Based Equations
Free-Space Gain in dBi
Number of Elements
Modeled
Calculated

Fig. 6

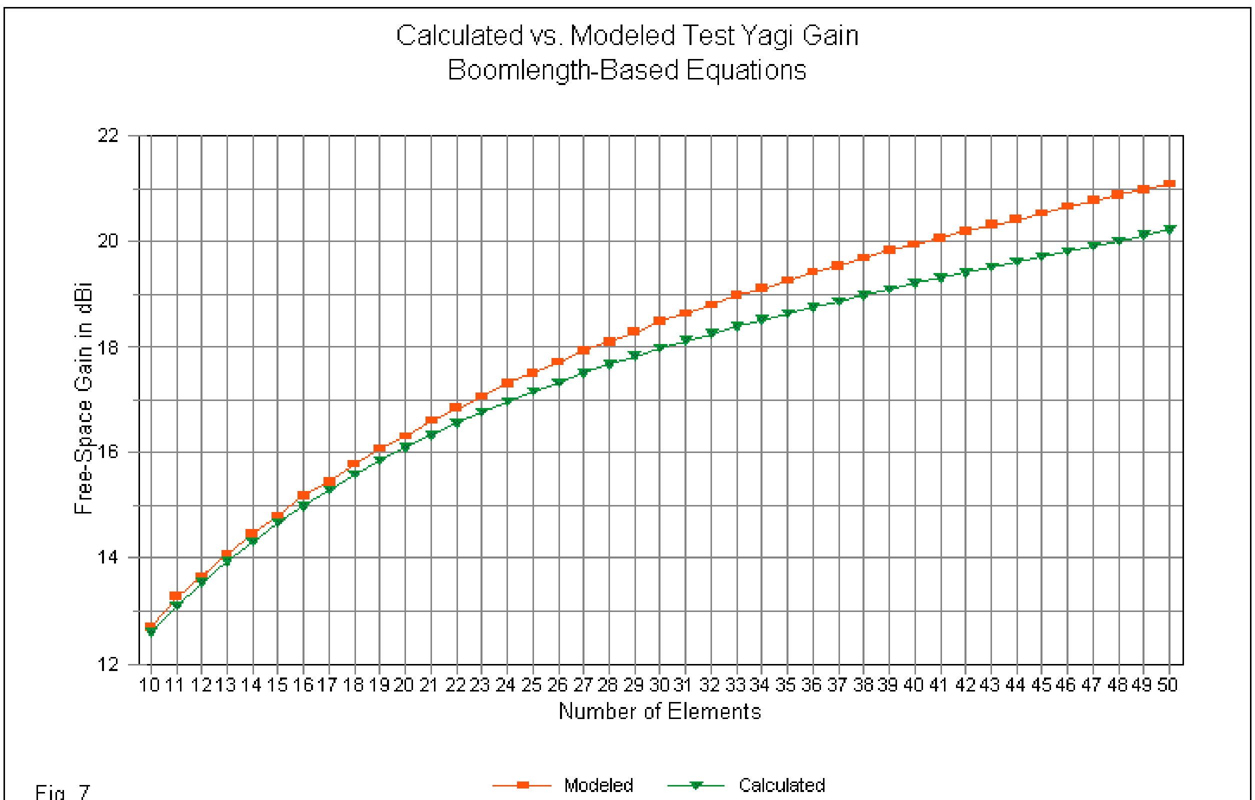

Calculated vs. Modeled Test Yagi Gain
Boomlength-Based Equations
Free-Space Gain in dBi
Number of Elements
Modeled
Calculated

Fig. 7

Beamwidth from Gain

The DL6WU design program (DL6WU-GG.EXE) makes use of a pair of equations to derive both the horizontal (E-plane) and vertical (H-plane) beamwidths from the array gain. The function of these estimates is primarily to provide further estimates of the required spacing of beams in horizontal and vertical stacks. Our question is how well they match up with the modeled data for each boomlength and element count in our 3 Yagi series.

The equation for the horizontal beamwidth uses the gain, which is adjusted for dBi in this version.

$$HBW = 30 - [\pi(G_{dBi} - 16.15)]$$

The corresponding equation to find the vertical beamwidth uses a presumed relationship between the horizontal and vertical beamwidths.

$$VBW = \left[\frac{HBW}{\cos\left(\frac{HBW}{2}\right)}\right]$$

As we have done with the other rules of thumb, we may use both graphs and tables to note the degree of coincidence (or non-coincidence) between calculated and modeled data. As usual, we begin with the DL6WU Yagi series. See **Fig. 8** and **Table 7**. Of first note is the fact that we obtain opposing trends. The horizontal values coincide at the shortest boomlengths, while the vertical values coincide best at the longest boomlengths.

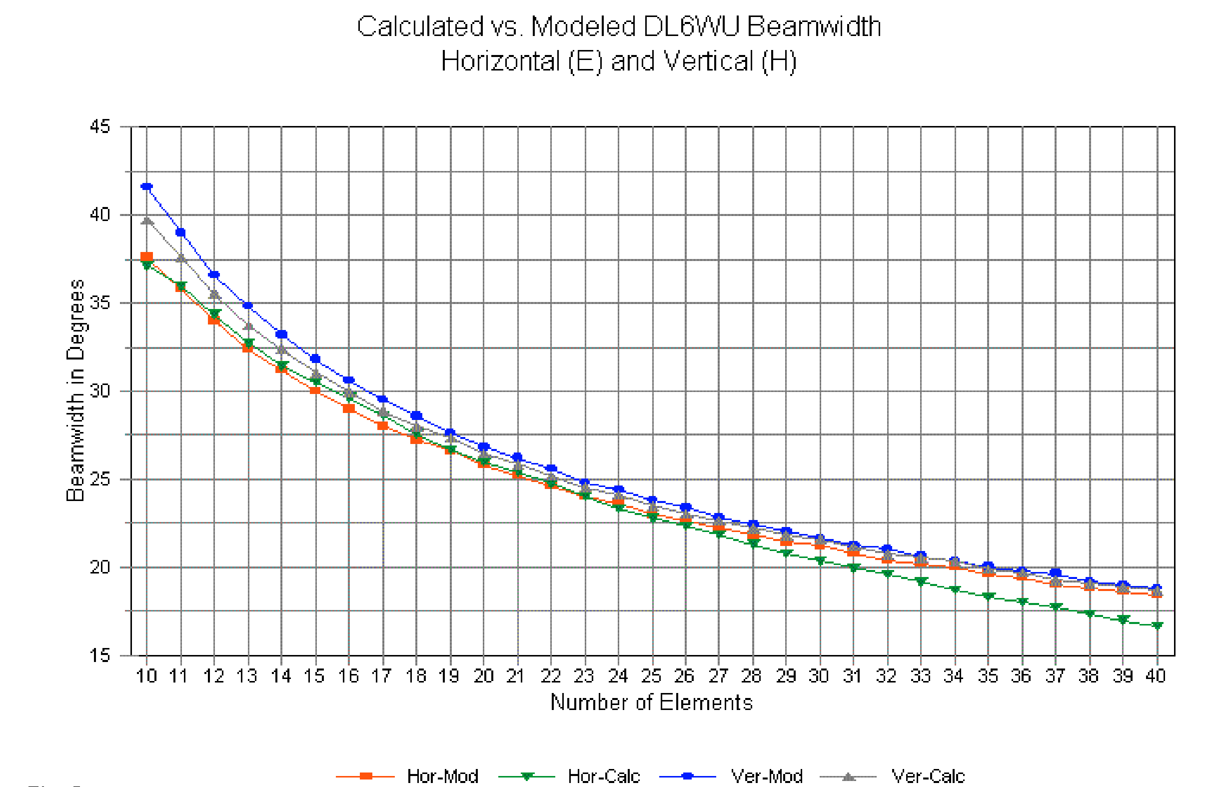


Fig. 8

DL6WU 40-Element Series			Beamwidth Calc.	
Table 7	HBW Calc		VBW Calc	
Elements	dl6wu-gg	% Error	dl6wu-gg	% Error
10	37.13142	1.261963	39.71904	4.735663
11	35.93761	-0.38291	37.62107	3.665312
12	34.36681	-1.06735	35.55352	2.943394
13	32.73319	-1.01788	33.73968	3.142655
14	31.44513	-0.77956	32.39329	2.490376
15	30.50265	-1.64791	31.05829	2.388138
16	29.62301	-2.10312	29.95411	2.156268
17	28.6177	-2.15845	28.85718	2.227585
18	27.58097	-1.38129	27.98466	2.198841
19	26.66991	-0.26214	27.3331	0.976454
20	25.97876	-0.68811	26.46802	1.254264
21	25.41327	-0.83923	25.82187	1.464362
22	24.75354	-0.62028	25.17795	1.676287
23	23.99956	0.001842	24.53617	1.075252
24	23.30841	1.251018	24.1095	1.204933
25	22.77434	0.990866	23.47119	1.400904
26	22.33451	1.188681	23.04677	1.532668
27	21.86328	1.54014	22.62322	0.781409
28	21.29779	2.358046	22.20053	0.89851
29	20.76372	3.064395	21.77867	1.016269
30	20.32389	4.310716	21.56805	0.14813
31	19.9469	4.276837	21.14742	0.248631
32	19.56991	4.241652	20.72759	1.314254
33	19.13009	5.592815	20.51796	0.399829
34	18.69027	7.00757	20.30853	0.450391
35	18.31328	7.026185	19.89024	0.551826
36	17.99912	7.783071	19.68138	0.602725
37	17.68496	7.435942	19.2642	1.743146
38	17.33938	8.423705	19.05588	0.756306
39	16.96239	9.654356	18.84774	0.807842
40	16.64823	10.52225	18.63978	0.859575

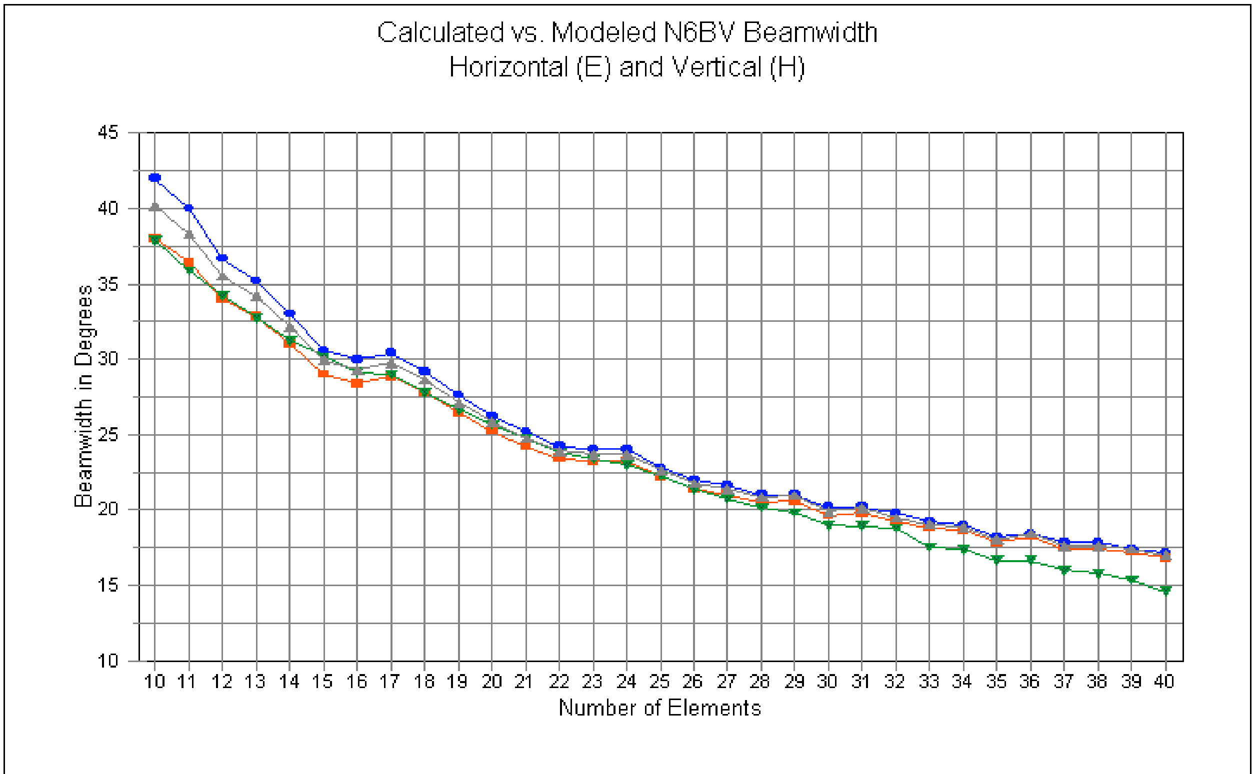
Due to the very wide range of boomlengths, the graph may mislead us into thinking that the divergence at the non-coincident ends of the lines is small. The table corrects this impression by showing the percentage of divergence. (The modeled values of horizontal and vertical beamwidth appear in the initial tables of modeled single unit performance.) At the shortest boomlengths, the vertical beamwidth is off (relative to modeled data) by nearly 5 percent. At the longest boomlengths, the equation yields values that are off by over 10 percent. However, in the middle of the range, for example, at the 25-element count, the equations are quite satisfactory for both beamwidth values.

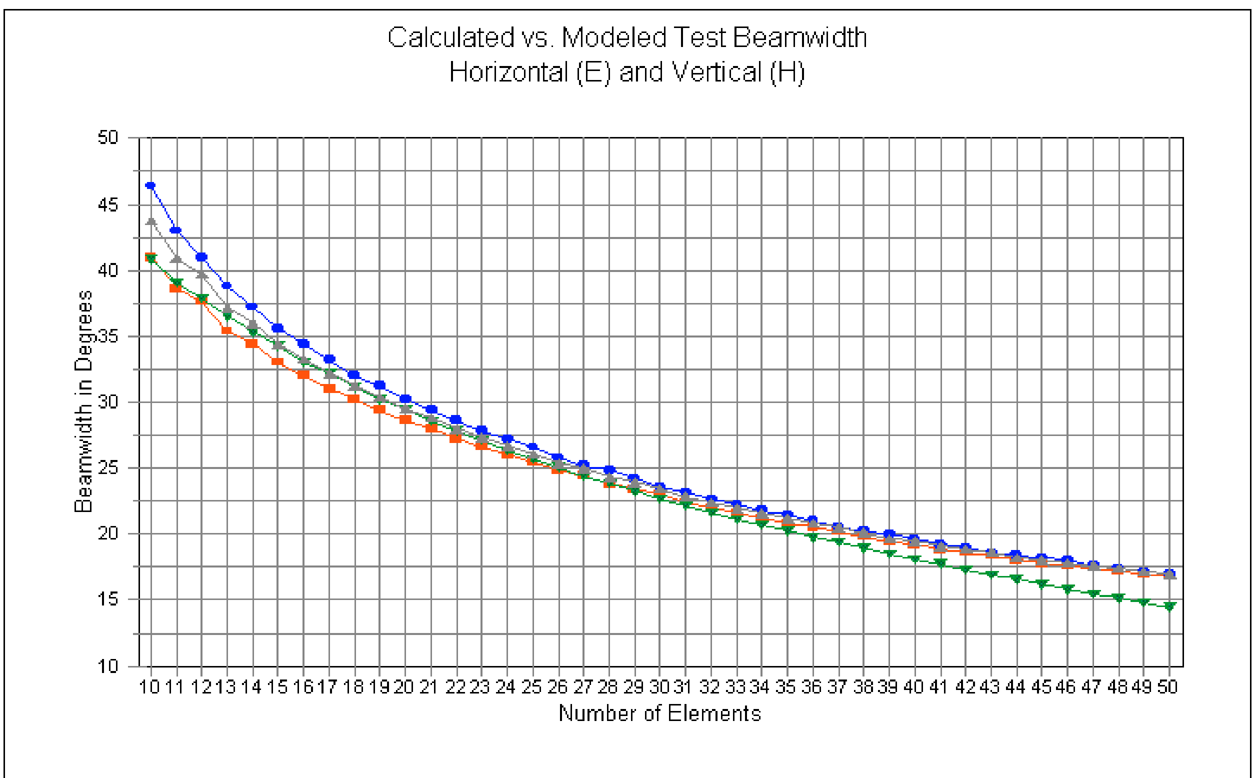
The superiority overall of the vertical beamwidth calculation is largely due to the fact that this calculation uses the modeled value of horizontal beamwidth as its basis within this exercise. The possible lack of rigor in the relationship of gain and beamwidth noted by Kraus remains operative in the horizontal beamwidth calculation that rests on gain (the modeled value). That possibility gives us reason to look at the data for the other Yagi series.

The data (**Table 8**) for the N6BV series of Yagis in fact confirms the general trends noted for the DL6WU Yagi series. However, the degree of divergence between modeled and calculated beamwidth values is not as great as in the DL6WU series. The maximum divergence for the vertical beamwidth is about 4.5 percent, while for horizontal beamwidth values, the divergence grows to only about 6 percent at the longest boomlengths.

N6BV 40-Element Series			Beamwidth Calc.	
Table 8	HBW Calc		VBW Calc	
Elements	dl6wu-gg	% Error	dl6wu-gg	% Error
10	37.8854	0.395473	40.18959	4.504685
11	35.90619	-1.06717	38.31692	4.392533
12	34.24115	-0.75332	35.55352	3.22466
13	32.73319	-3.02193	34.1911	2.950768
14	31.22522	-2.57522	32.17001	2.580016
15	30.21991	-3.71307	29.95411	2.156268
16	29.15177	-3.16923	29.2951	2.406199
17	28.96327	-3.72406	29.73415	2.239334
18	27.80089	-3.0883	28.63864	1.960148
19	26.66991	-2.71315	27.11645	1.783249
20	25.6646	-2.93875	25.82187	1.464362
21	24.75354	-1.94316	24.74986	1.81875
22	23.84248	-2.16139	23.8965	1.270051
23	23.37124	-1.88026	23.68373	1.335371
24	22.99425	-1.23208	23.68373	1.335371
25	22.20885	-1.152	22.62322	0.781409
26	21.39204	-1.06791	21.77867	1.016269
27	20.76372	0.096203	21.35764	1.13479
28	20.16681	-0.17816	20.83247	0.804169
29	19.82124	0.528454	20.93741	0.29896
30	19.00443	1.551203	19.89024	1.557345
31	18.94159	1.003568	20.09929	0.501052
32	18.78451	1.647787	19.47269	1.680841
33	17.52788	2.324603	19.05588	0.756306
34	17.3708	2.411077	18.84774	0.807842
35	16.64823	2.819393	18.01693	1.016121
36	16.64823	3.918529	18.43199	-0.17355
37	15.9885	4.39276	17.60254	1.121788
38	15.73717	4.705519	17.60254	1.121788
39	15.32876	5.212127	17.39559	0.025355
40	14.57478	6.116254	16.98218	1.282644

The question left by the table and **Fig. 9** is whether there is a reason for the greater coincidence between calculated and modeled values in the N6BV series. The N6BV series consists of individually optimized beams with a narrower operating passband, higher gain, and better general front-to-back ratio performance than the wide-band DL6WU series. As the early tables of modeled gain show, the N6BV series achieves higher gain for virtually every boomlength. However, the horizontal and vertical beamwidths in the N6BV are 16.8° and 17.2°, respectively, values that are smaller than the corresponding 40-element values for the DL6WU series by more than the gain differential would suggest. The DL6WU 40-element beamwidth values are 18.4° horizontal and 18.8° vertical for a gain differential of only about 0.7 dB.


Calculated vs. Modeled N6BV Beamwidth
Horizontal (E) and Vertical (H)
Beamwidth in Degrees
45
40
35
30
25
20
15
10
10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
Number of Elements
Hor-Mod
Hor-Calc
Ver-Mod
Ver-Calc
Fig. 9


Calculated vs. Modeled Test Beamwidth
Horizontal (E) and Vertical (H)
Beamwidth in Degrees
50
45
40
35
30
25
20
15
10
10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Number of Elements
Hor-Mod
Hor-Calc
Ver-Mod
Ver-Calc
Fig. 10

The ragged curves of the N6BV individually optimized Yagis give way to smoother curves for the test Yagi series that covers 10 to 50 elements and boomlengths from 1.5 λ to 14.5 λ. However, the lines in **Fig. 10** tell much the same story. The equations provide horizontal beamwidths that are coincident with the modeled data for shorter boomlengths and vertical beamwidths that are coincident at longer boomlengths. The error range, shown in **Table 9**, is even greater than for the DL6WU series: 6 percent for short boom vertical beamwidths and 16 percent for long boom horizontal beamwidths.

LB10-LB50 Yagi Series			Beamwidth Calc.	
Table 9	HBW Calc		VBW Calc	
Elements	dl6wu-gg	% Error	dl6wu-gg	% Error
10	40.83849	0.395473	43.77198	6.003877
11	39.01637	-1.06717	40.89845	5.138448
12	37.8854	-0.75332	39.71904	3.225052
13	36.5031	-3.02193	37.15905	4.416004
14	35.30929	-2.57522	36.01045	3.303358
15	34.27257	-3.71307	34.41731	3.436311
16	33.04734	-3.16923	33.28958	3.335632
17	32.19911	-3.72406	32.17001	3.201713
18	31.16239	-3.0883	31.28002	2.301736
19	30.21991	-2.71315	30.39489	2.648823
20	29.46593	-2.93875	29.51448	2.32264
21	28.55487	-1.94316	28.85718	1.881051
22	27.80089	-2.16139	27.98466	2.198841
23	27.10973	-1.88026	27.3331	1.708168
24	26.32434	-1.23208	26.68391	1.934099
25	25.69602	-1.152	26.03701	2.162279
26	25.0677	-1.06791	25.39235	1.605423
27	24.37655	0.096203	24.96378	0.946232
28	23.84248	-0.17816	24.32272	1.962281
29	23.27699	0.528454	23.8965	1.270051
30	22.64867	1.551203	23.47119	0.548796
31	22.17743	1.003568	22.83489	1.598927
32	21.64336	1.647787	22.41177	0.839883
33	21.10929	2.324603	21.98949	0.957301
34	20.70089	2.411077	21.56805	1.075427
35	20.22965	2.819393	21.14742	1.194372
36	19.72699	3.918529	20.83247	0.804169
37	19.35	4.39276	20.51796	0.399829
38	18.91018	4.705519	20.09929	0.501052
39	18.43894	5.212127	19.68138	1.618914
40	18.09336	6.116254	19.47269	0.653762
41	17.71637	6.11653	19.05588	0.756306
42	17.27655	7.660385	18.84774	0.807842
43	16.89956	8.878583	18.63978	-0.2134
44	16.5854	8.529192	18.22437	0.963697
45	16.20841	9.819544	18.01693	1.016121
46	15.8	11.3924	17.80965	1.068812
47	15.45443	12.58911	17.60254	-0.01441
48	15.14027	13.60434	17.39559	0.025355
49	14.79469	14.90608	17.1888	0.065134
50	14.44912	16.27009	16.98218	0.104938

Most interestingly, the longest N6BV and the longest test series Yagis have almost the same gain values and beamwidths. The N6BV gain is 21.06 dBi, while the test Yagi shows 21.1 dBi. The relevant N6BV beamwidths are 16.8° and 17.2°, while the corresponding test series values are 16.8° and 17.0°. If any problem exists for the horizontal beamwidth equation, it appears to be its large shift in outcome for small changes in gain, especially as the gain value reaches appreciable proportions.

Of course, if we use the modeled data as the basis for the gain value, then the remaining equation-based beamwidth calculations become superfluous. The same modeling that yields the array gain also yields data on the horizontal and vertical beamwidths. However, our goal here was not to legitimize the use of the equations so much as it has been to find their nature and limitations in terms of how well they replicate modeled behavior. Since we would normally use the beamwidth equations after first calculating the beam's gain apart from modeling, the values would show greater deviation from the modeled values based on the equation used to arrive at the gain value. Since the first set of equations for gain rest on beamwidth values and our latest set for beamwidth rest on gain, we cannot find a place to begin in the absence of a NEC model.

However, we can use the second boomlength-based gain equation to see what values we obtain. Let's examine a single case and compare the resulting calculated values with the modeled data. The DL6WU 40-element Yagi uses a 14.015 λ boom.

Method	Gain dBi	Hor. Beamwidth	Vert. Beamwidth
Modeled	20.40	18.4°	18.8°
Calculated	20.09	17.6°	17.8°

The results are quite different if we use the 40-element N6BV Yagi on its 13.95 λ boom.

Method	Gain dBi	Hor. Beamwidth	Vert. Beamwidth
Modeled	21.06	16.8°	17.0°
Calculated	20.08	17.7°	17.9°

Because the boomlengths are so similar, the boomlength-based equation yields nearly identical gain values, with resulting nearly identical beamwidth values. However, the N6BV calculated beamwidths are about as much too large as the DL6WU calculated values are too low, relative to modeled data for each Yagi.

How Good or Bad Are the Equations?

There is no single answer to our lead question. So let's look at two scenarios.

First, if we are doing preliminary planning for a Yagi installation, the boomlength-based gain equations and the beamwidth equations based on gain provide useful preliminary estimates that allow for initial decisions. For example, if a particular communications circuit requires a certain gain minimum, we can fairly quickly determine a boomlength that will yield that gain. Since the boomlength-based equation is conservative, especially for long-boom Yagis, then its value generally ensures that a well-designed Yagi of the resulting boomlength will achieve the desired gain. Using the beamwidth equations gives us a preliminary measure of the precision of aiming that we shall need to establish and maintain. However, the variability of designs with respect to their beamwidth suggests that we should subtract a degree or 2 from both beamwidths in setting preliminary aiming goals and specifications.

Note that in this exercise, I did not specify any particular design. The equations provide what Kraus has properly termed ballpark values and not values that apply to any particular Yagi design for a given boomlength.

For selecting a design to meet the overall objectives, the equations fail us. There are differences in the performance values for Yagis from each of the three sample series at the desired 14 λ boom selected for review. What those differences are emerges most ably from careful modeling (prior to range testing, of course). Whether the differences are significant depends upon the total set of specifications going into the design selection. The specifications may include such antenna parameters as front-to-back ratio (180° and/or worst-case), front-to-sidelobe ratio, operating passband, etc. There may also be a set of physical or mechanical specifications, and these may either favor or eliminate the higher element density of test-series Yagi of the desired boomlength and gain.

Used within their ballpark limits, the rules of thumb for calculating Yagi behavior may serve useful purposes. However, stretched beyond those limits, they quickly become more misleading than helpful. Only a few decades ago, the rule-of-thumb equations were almost all that amateur radio Yagi designers had to use as calculating tools. Computer modeling and optimization has largely replaced the rules of thumb with much more exacting analytical and design tools.

In general, thumbs are useful, but fingers are much more sensitive.